2018

(April)

STATISTICS

(Honours)

Survey Sampling and Non-parametric Inference)

[STEH–62(TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* What is circular systematic sampling? Write the procedure of drawing such type of samples. How does it differ from linear systematic sampling? 4

(b) If the population consists of a linear trend, $Y_i = i$, $i = 1, 2, \cdots, k$, then prove that

$$\text{var}(\bar{y}_{st}) \leq \text{var}(\bar{y}_{sys}) \leq \text{var}(\bar{y}_n)_R$$ 8

2. *(a)* In a simple random sampling without replacement, show that the regression estimator of mean is a biased estimator. Obtain the expression for its approximate variance.

(b) Prove that in simple random sampling without replacement, for large n an approximation to the variance of $\hat{R}$ is given by

$$V(\hat{R}) = \frac{1-f}{n\bar{X}} \sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{N-1}$$

where $f = \frac{n}{N}$ is the sampling fraction and $\hat{R}$ is the estimator of ratio $R = \frac{Y}{X}$.

UNIT—II

3. *(a)* What do you mean by cluster sampling? Give an example of it. Write the advantages of cluster sampling.

(b) If n clusters are selected from N clusters by simple random sampling without replacement, then show that $\bar{y}_c$ is an unbiased estimator of $\bar{Y}$ with variance

$$V(\bar{y}_c) \simeq \frac{1-f}{nM} S^2 [1 + (M-1)\rho]$$

where $\bar{y}_c$ is the cluster mean, $\bar{Y}$ is the population mean/element, ρ is the intra-cluster correlation coefficient and M is the cluster size.

(Continued)

4. *(a)* For a single-stage cluster sampling with equal size cluster, under SRSWR, obtain the estimate of population total along with its variance. 7

(b) Write a note on relative efficiency of cluster sampling. 4

UNIT—III

5. In two-stage sampling, if n units are selected from N primary units and from each selected primary unit, if m units are selected from M secondary units by SRSWOR scheme, then show that sample mean $\bar{y}$ is an unbiased estimator of $\bar{Y}$ and the variance of this estimator is

$$V(\bar{y}) = \frac{1-f}{n} S_b^2 + \frac{M-m}{M} \frac{S_w^2}{nm}$$ 11

6. *(a)* What is two-stage sampling? Give an example of it. Write the advantages and uses of two-stage sampling. 4

(b) Obtain unbiased estimator of $V(\bar{y})$ given in Question No. **5**. 7

(Turn Over)

UNIT—IV

7. *(a)* What do you mean by 'order statistics'? What purpose does it serve?

(b) Define cumulative distribution function and probability density function of a single-order statistic. Also deduce their expressions.

8. Obtain the expression for joint p.d.f. of two-order statistics and hence obtain the distribution of range.

UNIT—V

9. *(a)* Explain what is meant by non-parametric methods. Distinguish between parametric and non-parametric approaches to the theory of statistical inference. $2+5^{1}$

(b) Derive sign test for two-sample problem.

10. Write short notes on the following : $5\frac{1}{2}\times 2=11$

(a) Mann-Whitney test

(b) Kolmogorov-Smirnov test (one sample)

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